International Journal of Applied Mathematics
& Statistical Sciences (IJAMSS)
ISSN(P): 2319-3972; ISSN(E): 2319-3980
Vol. 6, Issue 5, Aug – Sep 2017; 95-98

International Academy of Science, Engineering and Technology
Connecting Researchers; Nurturing Innovations

NEW SHRINKAGE ESTIMATOR USING WEIBULL DISTRIBUTION

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ABSTRACT

The parameter of Weibull distribution was estimated by Bayes, and maximum likelihood estimation. Using a linear combination between maximum likelihood method and Bayes method, we obtained new shrinkage estimator. Then simulation study will be used to compare between all estimators, to find the best (least mean square error) with different sample size and Mathlab program.

KEYWORDS: Shrinkage Estimator, Bayes Estimator, Maximum Likelihood Estimator

INTRODUCTION

In 2008 Baklizi and Ahmed [5] studied three classes of point estimators, unrestricted estimator, shrinkage estimator and shrinkage preliminary test estimator. Then, mean squared errors are derived and compared. Morris, Baggerly and Coombes [6] developed a new Bayesian estimation procedure, that quantifies prior information about two characteristics, yielding a nonlinear shrinkage estimator with efficiency advantages over the MLE. The shrinkage estimators of the parameter exponential distribution have been studied by Chiou and Han [7], they introduced the usual preliminary test estimators of the threshold parameter of the exponential distribution, in censored samples. We studied in this paper shrinkage estimator, $\tilde{\theta}$, Bayes and Maximum likelihood estimators. We compared between estimators, using a simulation study to find the best estimator depending on less mean square error, MSE.

The estimator of parameter Weibull distribution was estimated (Alkutubi [1,2,3,4]), suppose $t_1,\ldots,t_n$ be a random sample of size n with probability (y) and sity function f (t, θ). Using Bayesian and Maximum likelihood (MLE) methods to obtain these estimators, the probability density function of weibull case is $f(t,\theta,c)=\frac{c}{\theta}\left(\frac{t_i}{\theta}\right)^{c-1} exp\left(\frac{-t}{\theta}\right)^{c}$, and

$\hat{\theta}_M=\frac{\sum t_i}{\sqrt[c]{cn}}$ is the maximum likelihood estimator and

$\hat{\theta}_B=\frac{\sum t_i}{\left(\frac{n-1}{c}\right)}$ is the Bayes estimator.

MATERIALS AND METHODS

Shrinkage Estimators $\widehat{\theta_B}$ and $\widehat{\theta_M}$ Between Bayes and MLE

We obtain shrinkage estimator $\tilde{\theta}$ From Bayesian and maximum likelihood estimators by linear combination (see Alkutubi and Ibrahim 2009) such that,

$$\tilde{\theta}=p\widehat{\theta_M}+(1-p)\widehat{\theta_B}$$

The value of p which minimizes $MSE(\tilde{\theta})$ is,

$p = \frac{n^2+n-1}{2n^2-2n-2}$

Then the shrinkage estimator $\tilde{\theta}$ between Bayes and MLE is given by

$$\tilde{\theta} = \left(\frac{n^2+n-1}{2n^2-2n-2}\right)\widehat{\theta_M} + \left(1 - \frac{n^2+n-1}{2n^2-2n-2}\right)\widehat{\theta_B}$$

Simulation Study

In this study, we choose samples sizes n=30, 60, 90, with parameter value θ = 0.5,1, 1.5 and R=1000 of replication. Using mean square error (MSE) and mean percentage error (MPE) to compare between all estimators $\tilde{\theta}$, $\hat{\theta}_M$, and $\hat{\theta}_B$, where

$$MSE(\tilde{\theta}) = \frac{\sum_{i=1}^{R}\left(\hat{\theta}_i - \theta\right)^2}{R}, \quad \text{MPE}\left(\hat{\theta}\right) = \frac{\left[\sum_{i=1}^{1000}\frac{\left|\hat{\theta}_i - \theta\right|}{\theta}\right]}{R}$$

The simulation program is written by Mat lab program. and the results are introduced and tabulated in table 1 and table 2, for the MSE, MPE of all estimators for all sample sizes and θ values respectively.

RESULTS AND DISCUSIONS

Table 1: MSE of Estimators

n	θ	$\tilde{\theta}$	$\widehat{\theta_M}$	$\widehat{\theta_B}$
30	0.5	0.0328	0.0330	0.0324
	1	0.0327	0.0328	0.0329
	1.5	0.0326	0.0327	0.0325
60	0.5	0.0226	0.0227	0.0229
	1	0.0225	0.0227	0.0227
	1.5	0.0222	0.0222	0.0222
90	0.5	0.0135	0.0137	0.0139
	1	0.0086	0.0087	0.0087
	1.5	0.0041	0.0047	0.0045

Table 2: MPE of Estimators

n	θ	$\tilde{\theta}$	$\widehat{\theta_M}$	$\widehat{\theta_B}$
30	0.5	0.0210	0.0210	0.0214
	1	0.0227	0.0228	0.0229
	1.5	0.0226	0.0227	0.0225
60	0.5	0.0126	0.0127	0.0129
	1	0.0125	0.0127	0.0127
	1.5	0.0122	0.0122	0.0122
90	0.5	0.0035	0.0037	0.0039
	1	0.0026	0.0027	0.0027
	1.5	0.0001	0.0007	0.0005

In Table 1 and 2, we can see the shrinkage estimators $\tilde{\theta}$, is the best (less mean square error) shrinkage estimators from all estimators and for all sample size and parameter value. The effect of sample size on the mean square error and the mean percentage error of all estimators, decreases as sample size n increases.

CONCLUSIONS

The generator shrinkage estimators $\tilde{\theta}$ is the best estimators (less mean square error). The effect of sample size on the mean square error and the mean percentage error of all estimators refer to the MSE, MPE decreases as n increases.

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